Problems ♠–4

Extended deadline: *Due in DSC 235 by 12 noon, Friday, February 16, 2018*

Problem A: Let $m > 1$ be a natural number. Prove that there exist an integer $k \geq 2$ such that

- k is divisible by m, and
- the decimal expansion of k has zeros and ones only.

Problem B: Let $a, b, x_0 \geq 1$ be natural numbers and for $n \geq 1$ let

$$x_n = ax_{n-1} + b.$$

Show that there are infinitely many composite numbers in the sequence $\left(x_n\right)_{n=1}^{\infty}$.

Valentine's Day Special: Every correct solution will be awarded a box of chocolates. And you know how it is with *a box of chocolates. You never know what you're gonna get.*

Rules:

- The competition is open to all *undergraduate* UNO students.
- The problems are selected and graded by the POW Committee consisting of Professors Andrzej Rosłanowski and Vyacheslav V. Rykov. However, please always submit your solutions to Andrzej Rosłanowski in DSC 235 or to his mailbox.
- Every nontrivial step/claim in your solution must justified. There are no partial credits, so rather err on the side of caution and provide more explanations than less. (Needless to say, they should be be written clearly and legibly.)
- The winners of Spring 2018 edition of POW will be determined at the end of the semester based on the number of correct solutions submitted.
- Problems will be posted by Friday 5pm and the solutions are due by the following Friday 12 noon.

Prizes:

- Cash rewards for the winners of 1st, 2nd and 3rd place are \$400, \$200 and \$100, respectively. In the case of multiple persons receiving the same score the prizes will be suitably divided by the Committee.
- Everybody scoring in the POW Competition will be allowed to participate in the grand finale:

  $\frac{\pi}{2}$ *Mathematical Competition.*

  The grand finale will take place on Saturday, April 21, 2018 with grand prizes of \$300, \$400, and \$600.